BALKANJM 03 (2015) 185-193

Contents lists available at BALKANJM

BALKAN JOURNAL OF MATHEMATICS

journal homepage: www.balkanjm.com

A Decision Support Model Applying Fuzzy AHP for Location Selection

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ARTICLE INFO

Article history:
Received: January 18, 2015
Accepted: February 25, 2015
Available online: June 8, 2015

Keywords:
MCDM
Location Selection
Fuzzy Sets

ABSTRACT

Location selection is one of the most important decision making process which requires to consider several criteria based on the mission and the strategy. This study's object is to provide a decision support model in order to help the bank selecting the most appropriate location for a bank's branch considering a case study in Turkey. The object of the bank is to select the most appropriate city for opening a branch among three alternatives in the South of Turkey. The model in this study was consisted of four main criteria which are Demographic, Socio-Economic, Banking and Trade Potential which represent the bank's mission and strategy. Because of the multi-criteria structure of the problem and the fuzziness in the comparisons of the criteria fuzzy AHP is used.

1. Introduction

Location selection has a strategic importance for many companies .The general procedure for making location decisions usually consists of the following steps: Decide on the criteria that will be used to evaluate location alternatives; select the criteria that are important; develop location alternatives and select the alternatives evaluated [20]. Selecting a location is very important decision for firms because they are costly and difficult to reverse. A poor choice of location might result in excessive transportation costs, lots of qualified labor, competitive advantage or some similar condition that would be detrimental to operations [20]. Each organization should consider meaningful criteria for location selection suitable to its mission and strategy in order to make an efficient and effective strategic decision. The location decision may differ with regard to type of business. Thus, the factors considered vary from business to business but it is emphasized that the objective of the decision is to maximize the benefit of location of the firm [9].

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Location selection is a multi-criteria decision because it requires to take into consideration both qualitative and quantitative factors. The literature including bank branch location has also shown that that the selection process is a multi-staged process having different criteria in each level. In the literature, several approaches can be seen to handle multi-criteria problem. The analytic hierarchy process (AHP) developed by Saaty [19] is used methodology for his type of problems [12], [1]. AHP allows to structure multi criteria problem hierarchically and to combine the results obtained at each level of the hierarchy but cannot reflect the human thinking style which is uncertain and imprecise Therefore, fuzzy AHP is used to obtain the judgments for the decision making process. In the literature, different approaches to fuzzy AHP such as Buckley [3], Chang [7], Leung and Cao [14] and Buckley et al.[4] can be found. In this study, Chang's extent analysis method is used to compare the criteria. The authors have used this fuzzy approach to compare the catering services companies in Turkey [11], to develop a framework for quality function deployment (QFD) planning process using analytic network approach [14], to valuate machine tool alternatives [1], for the selection among computer integrated manufacturing systems [3], for the operating system selection using fuzzy replacement analysis and analytic hierarchy process [21].

2. Fuzzy Numbers

A fuzzy number $\widetilde{M}$ is a convex normalized fuzzy set $\widetilde{M}$ of the real line R such that it exists such that one $x_0 \in R$ with $\mu_{\widetilde{M}}(x_0) = 1$, $\mu_{\widetilde{M}}(x)$ is piecewise continous. x_0 is called mean value of $\widetilde{M}$.

It is possible to use different fuzzy numbers according to the situation. Generally in practice triangular and trapezoidal fuzzy numbers are used. In applications, it is often convenient to work with triangular fuzzy numbers (TFNs) because of their computational simplicity and they are useful in promoting representation and information processing in a fuzzy environment. In this paper, TFNs are used.

Triangular fuzzy numbers can be expressed as (l, m, u). These parameters indicate the smallest possible value, the most promising value and the largest possible value that describe a fuzzy event.

There are various operations on triangular fuzzy numbers. But here, three important used in this study are illustrated. If we define, two positive triangular fuzzy numbers (l_1, m_1, u_1) and (l_2, m_2, u_2) then

$$(l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad \textbf{(1)}$$

$$(l_1, m_1, u_1) \,.\, (l_2, m_2, u_2) = (l_1 . l_2, m_1 . m_2, u_1 . u_2) \quad \textbf{(2)}$$

$$(l_1, m_1, u_1)^{-1} \approx \left(1/u_1, {}^{1}/_{m_1}, {}^{1}/_{l_1}\right) \quad \textbf{(3)}$$

3. Extent Analysis Method On Fuzzy AHP

In this study, Chang's [8] extent analysis method on fuzzy AHP, therefore triangular fuzzy numbers (TFN) are used. Triangular fuzzy numbers are represented as l/m, m/u, (or (l, m, u) in which l, m and u refer to, respectively, the lower value, modal value and upper value.

Let $X = \{x_1, x_2, ..., x_n\}$ an object set and $G = \{g_1, g_2, ..., g_n\}$ a goal set respectively. Then each object is taken and extent analysis for each goal is performed respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{gi}^1, M_{gi}^2, ... M_{gi}^m \qquad i\_1,2,...n$$

Where M_{gi}^j $(j = 1,2,...m)$ all are TFNs. The steps of Chang's [8] extent analysis can be given as following:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \qquad \textbf{(4)}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed such as:

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \qquad \textbf{(5)}$$

to obtain $\left[\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1}$ the fuzzy addition operation of M_{gi}^j (j=1,2,…,m) values is performed such as :

$$\sum_{i=1}^{m} \sum_{j=1}^{m} M_{gi}^j = \left(\sum_{i=1}^{n} l, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \qquad \textbf{(6)}$$

and then inverse of the vector above is computed , such as:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \qquad \textbf{(7)}$$

Step 2*:* As $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers , the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as

$$V(M_2 \geq M_1) = \sup_{y \geq x} \left[\min \left(\mu_{M_1}(x), \mu_{M_2}(y) \right) \right] \qquad \textbf{(8)}$$

and can be expressed as follows ($m_2 \geq m_1, l_1 \geq u_2, otherwise$ respectively):

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1 \\ 0 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - u_1)} \end{cases} \quad (9)$$

Where d is the ordinate of the highest intersection point D between μ_{M_1} and $\mu_{M_{21}}$. To compare M_1 and M_2 , we need both the values $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy $M_i (i = 1,2,...k)$ numbers can be defined by (i=1,2,…,k)

$$V(M \geq M_1, M_2, ..., M_K) = V[(M \geq M_1 \wedge M \geq M_2 \wedge ...M \geq M_K] = \min V(M \geq M_i) \quad \textbf{(10)}$$

Assume that $d(A_i) = \min V(S_i \geq S_k)$ for k= 1,2,…,n ; $k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), ..., d'(A_i))^T \quad \textbf{(11)}$$

where A_i are n elements.

Step 4: Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), ..., d(A_n)) \quad \textbf{(12)}$$

where W is a non fuzzy number.

4. Developing A Decision Support Model For Bank Branch Locaion Selection

As mentioned above, the aim of this study is to select the best bank branch location among the alternatives using fuzzy AHP to determine the weights of main criteria to evaluate the potential locations considering weights of the criteria and to rank them. The object of the bank is to decide which city among three alternatives in the South part of Turkey a branch should be opened based on its vision and strategy.

Firstly, the criteria for the selection decision were identified. Considering the studies in the literature which are [2],[16-18],[23] and the discussions with the bank's managers in different areas, many criteria were determined as demographic (C1), banking (C2), sectoral employment(C3) and trade potential (C4).

Once the model was constructed, a questionnaire form was established to obtain the bank managers' pair wise comparisons for the main criteria and for evaluating the candidate cities. In the form, three bankers indicated their pair wise comparisons to obtain the weights of the main criteria and the sub-criteria using the linguistic scale [12] which is presented in Table 1 and the associated fuzzy pair wise comparisons are also given in Table 2. The fuzzy values are the mean value of the evaluations.

Linguistic scale	Triangular Fuzzy Numbers
Absolutely more important (A)	(5/2, 3, 7/2)
Very strongly more important(VS)	(2, 5/2, 3)
Strongly more important (S)	(3/2, 2, 5/2)
Weakly more important (W)	(1, 3/2, 2)
Equally important (E)	(1/2, 1, 3/2)
Just equal (J)	(1, 1, 1)

Table 1 Linguistic Variables Used for the Evaluation of the Criteria

	C1	C2	C3	C4
C1	(1,1,1)	(0.86,1.17,1.16)	(0.67,1,1.5)	(0.33,0.39,0.49)
C2	(0.64,0.85,1.16)	(1,1,1)	(2.5,3,3.5)	(0.95,1.33,1.83)
C3	(0.67,1,1.5)	(0.29,0.33,0.4)	(1,1,1)	(0.4,0.5,0.67)
C4	(2.04,2.56,3)	(0.55,0.75,1.05)	(1.5,2,2.5)	(1,1,1)

Table 2 Fuzzy Pair wise Comparison Matrix

After forming fuzzy pair-wise comparison matrix , weights of all criteria are determined. According to th FAHP Method, firstly synthesis values must be calculated. Using the Table 2,synyhesis values respect to main goal are calculated like in Eq. (4):

$$S_1 = (2.86,3.56,4.55) \otimes \left(\frac{1}{23.18}, \frac{1}{18.88}, \frac{1}{15.59} \right) = (0.12,0.29,0.29)$$

$$S_2 = (5.09,6.18,7.49) \otimes \left(\frac{1}{23.18}, \frac{1}{18.88}, \frac{1}{15.59} \right) = (0.22,0.32,0.48)$$

$$S_3 = (2.56,2.83,3.56) \otimes \left(\frac{1}{23.18}, \frac{1}{18.88}, \frac{1}{15.59} \right) = (0.11,0.15,0.23)$$

$$S_B = (5.08,6.31,7.58) \otimes \left(\frac{1}{23.18}, \frac{1}{18.88}, \frac{1}{15.59} \right) = (0.21,0.33,0.49)$$

These fuzzy values are compared by using Eq.(9) and these values are obtained:

$$V(S_1 \geq S_2) = \frac{0.22 - 0.29}{(0.19 - 0.29) - (0.32 - 0.22)} = 0.35$$

$$V(S_1 \geq S_3) = 1$$

$$V(S_1 \geq S_4) = \frac{0.21 - 0.29}{(0.19 - 0.29) - (0.33 - 0.21)} = 0.32$$

$$V(S_2 \geq S_1) = 1$$

$$V(S_2 \geq S_3) = 1$$

$$V(S_2 \geq S_4) = \frac{0.21 - 0.48}{(0.32 - 0.48) - (0.33 - 0.21)} = 0.96$$

$$V(S_3 \geq S_1) = \frac{0.12 - 0.23}{(0.15 - 0.23) - (0.19 - 0.12)} = 0.73$$

$$V(S_3 \geq S_2) = \frac{0.22 - 0.23}{(0.15 - 0.23) - (0.32 - 0.22)} = 0.06$$

$$V(S_3 \geq S_4) = \frac{0.21 - 0.23}{(0.15 - 0.23) - (0.33 - 0.21)} = 0.10$$

$$V(S_4 \geq S_1) = 1$$

$$V(S_4 \geq S_2) = 1$$

$$V(S_4 \geq S_3) = 1$$

The the priority weights are calculated by using Eq.(11)

d' $(C1)$ =min (0.35, 1, 0.32) =0.32
d' $(C2)$ =min(1, 1, 0.96) = 0.96
d' $(C3)$ =min(0.73, 0.06, 0.10) = 0.06
d' $(C4)$ =min(1, 1, 1)=1

Priority weights form W' = (0.32, 0.96, 0.06, 1) vector. After the normalization of these values priority weights respect to main goal are calculated as (0.13, 0.41, 0.03, 0.43).

After the determining the weights of the criteria, FAHP method is considered for the comparison of each alternative for each criterion. The fuzzy pair-wise matrices of the comparisons and the weight of the criteria calculated are given as in Tables 3,4,5 and 6.

C1	A1	A2	A3	W_{C1}
A1	(1,1,1)	(0.67, 1, 1.5)	(0.54, 0.75, 1.1)	0.28
A2	(0.67, 1, 1.5)	(1,1,1)	(0.4,0.5,0.6)	0.21
A3	(0.91,1.33,1.85)	(1.5,2,2.5)	(1,1,1)	0.51

Table 3 Comparison of The Alternatives For The Demographic Criteria(C1)

C2	A1	A2	A3	W_{C2}
A1	(0.33, 0.33, 0.34)	(0.28,0.33,0.39)	(0.25,0.33,0.42)	0.66
A2	(0.29,0.33,0.4)	(0.33,0.33,0.34)	-	0.16
A3	(0.24, 0.32, 0.43)	-	(0.33,0.33,0.34)	0.19

Table 4 Comparison of The Alternatives For The Banking Criteria(C2)

C3	A1	A2	A3	W_{C3}
A1	(0.33, 0.33, 0.34)	(0.27, 0.33, 0.40)	(0.28, 0.33, 0.39)	0.35
A2	(0.29, 0.32, 0.4)	(0.33, 0.33, 0.34)	(0.21, 0.32, 0.47)	0.33
A3	(0.28, 0.32, 0.39)	(0.21, 0.32, 0.47)	(0.33, 0.33, 0.34)	0.32

Table 5 Comparison of The Alternatives For The Banking Criteria(C3)

C4	A1	A2	A3	W_{C4}
A1	(1,1,1)	-	(0.95,1.25,1.59)	0.22
A2	-	(1,1,1)	(1.5,2,2.5	0.42
A3	(0.95,1.25,1.59)	(0.4,0.5,0.67)		0.36

Table 6 Comparison of The Alternatives For The Trade Potential Criteria(C4)

These matrices are used to obtain the weights of each alternative for each criterion which are given in Table7.

	A1	A2	A3
C1	0.28	0.21	0.51
C2	0.66	0.16	0.19
C3	0.35	0.33	0.32
C4	0.22	0.42	0.36

Table 7 The Weights of The Criteria

The final scores are obtained by multiplication the weight of each criterion by the alternatives' weights obtained for each criterion . For example, the final score for A1 is calculated as:

0.28*0.13+0.66*0.41+0.35*0.03+0.220.43=0.41

All the results are given in Table 8:

	A1	A2	A3
Scores	0.41	0.28	0.25

Table 8 The alternatives' scores

It's obviously seen that alternative A1 which has the highest priority weight is selected as a best location for the bank branch and the banking criteria which has the highest weight is the first criteria considered by the evaluators.

5. Conclusion

Branches have a strategic importance on a bank's performance and competitiveness [12],[18-19] and the banks must identify meaningful criteria for their location selection considering their missions and strategies. In this paper we have studied a problem of bank branch location selection. A linguistic decision process is proposed to solve the multi-crtieria decision making problem under fuzzy environment. Considering the fuzziness in the linguistic terms, the variables are used to assess the weights of all criteria and the ratings of each alternative in terms of every criterion.

In this study, FAHP is used. Decision makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision making process. By using fuzzy AHP, uncertainty and vagueness from subjective perceptions and experiences of decision makers can be effectively represented and reached to a more effective decision.

Finally, we admit that the order of the alternatives is A1>A2>A3. For the decision makers, the best opinion was A1 because it is the best solution specially considering the banking criterion.

It is well known, the applicability of FAHP is due to its simplicity, ease of use and great flexibility. In this way, it is possible to be integrating with other different techniques to obtain the aggregation of the preferences, for instance arithmetic mean, harmonic mean are possible.

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