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Mathematical and Statistical Modeling of the Musical Compositions

Filiz Gulsoy[*a], Ibrahim Guney[b] and Ertugrul Ozdamar[c]

[a] *Department of Mathematics, Uludag University, Bursa, Turkey*
[b] *Department of Mathematics, Bitlis Eren University, Bitlis, Turkey*
[c] *Department of Mathematics, Bahcesehir University, Istanbul, Turkey*

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ABSTRACT

In this paper, we apply coding method to musical notes so that every note corresponds to a point in the space R^4. We use a regression method with a fitted manifold (a curve or a hypersurface), which we call it the shape of the composition. Then, based on the statistical results of significance of the manifold, we give some differential geometric properties of the manifold.
As an application of the above correspondence between the composition and the manifold, we give an example, in which the composition is Viertes symphony L.Van Beethoven, Op.60. We show that the symphony corresponds to a hypersurface in the space R^4 and give some curvature properties and plot sectional surfaces of the hypersurface. Statistically, we test the notes of symphony's corresponding hypersurface. We pose that the problem of finding the characteristics of the compose the musical composition of the corresponding manifolds.

1. Introduction

The Scholarly literature on this subject can be classified into two main groups. The first group, for which data is obtained from some software, can be modeled statistically. The second analyzes the data of musical compositions as [1]. Of course I. Xenakis's book, Formalized Music [2], is a good explanatory one for modeling musical compositions. Taking time parameter sequential cumulative time of musical notes we find that for L.Van Beethoven Vierte Symphony, the data is fits well for stepwise regression model and our data is obtained as the notes corresponds to the points in a Lorentzian space; M.D. McIlroy modeled the flow

[*] Corresponding author: *E-mail*: gfiliz@uludag.edu.tr (F. Gulsoy).
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music as a power series of sin, cos, square root, exp, arctan [3]. In the following part of this paper we use the same power series for stepwise regression modeling to get statistical manifold modeling of the data. R.C. Red used 12 note sequence and gave some combinatorial properties for cords [7]. However in this paper we use more than 57 note sequences and modeling a musical-compositions as manifolds. The books we used throughout the paper for basic notions of differential geometry and statistics: Elementary Differential geometry [8] and Regression Analysis [9].

2. The model

Five-tuples of real numbers defines a point in R^5, as $(x_1, x_2, x_3, x_4, x_5) \in R^5$, where x_1, x_2, x_3, x_4, x_5 represent the time axis, the numerical value of notes, the intensity of sound, timbre, duration of the playing time of the notes, respectively. In this paper, x_2 the numeric value of the notes varies in the interval [-20,37] as indicated in Table 1.

Table 1. Corresponding x_2 - values of notes

20	…	0	1	2	3	4	5	6	7	8	9	10	11	…	37
	…	*do*		*re*		*mi*	*fa*		*sol*		*la*		*si*	…	

Table 2. Duration time for each note

𝅝	𝅗𝅥	♩	𝅘𝅥𝅯	𝅘𝅥𝅰	𝅘𝅥𝅱
32	16	8	4	2	1
𝄻	𝄼	𝄽	𝄿	𝅀	𝅁

x_5 4s the duration time for the notes and it varies in the closed interval [0,32] as indicated in Table.2. x_5 is canceled since playing time and numerical values of notes can be stated in axes x_1 and x_2. Thus, a musical composition actually can be represented in the 4-dimensional space R^4. We can take the space R^4 as Minkowski space, where x_1 for time like, and x_2, x_3, x_4 for space like components. In the beginning Xenakis book [2] gives the vector (c_a, h_t, g_j, v_k) as c_a :timbre or instrumental family, h_t :pitch of the sound, g_j :intensity of sound, v_k :duration of the sound. In this paper, we add time parameter as a sequential cumulative duration of the sounds, so x_1 :sequential cumulative time, x_2 :pitch of the sound,

x_3 :duration of the sound, x_4 :timbre or instrumental family, x_5 :intensity of sound. By applying stepwise regression model, we take the notes as a dependent variable and time is an independent variable. The model we apply is:

$$x_2 = f(x_1, x_3, x_4, x_5, \exp(x_1), \sin(x_1), x_1^2, ...etc.) + \varepsilon$$

In this model, x_2 represents the numerical values of notes as given in Table 1, x_5 attaches a numerical value of notes as given in Table 2 so that the duration of the notes can be calculated as adding them together. In a symphony; we can sort out the notes from the beginning to the end; suppose *n*-th note P_n has x_2-coordinate denoted as $(P_n)_2$ and x_5-coordinate denoted as $(P_n)_5$; inductively; x_1–coordinate denoted as $(P_n)_1$ is computed as the following;

$$(P_n)_1 = (P_{n-1})_1 + (P_n)_5$$

Thus, the dependent variable x_2 and the independent variables x_1, x_3, x_4, x_5 are known. Applying regression analysis to the data obtained as above gives a model surface; which has an equation $x_2 = f(x_1, x_3, x_4, x_5, \exp(x_1), \sin(x_1), x_1^2, ...etc.)$. Specially; if x_5 constant (in the case of constant intensity of voice), x_4 :constant (in the case of just one instrument), then we get a curve 1-manifold attached to the symphony; (constant timbre and constant intensity curve; CTI) Taking x_1-axis time like and the others space-like we get a modeling for the symphony in Minkowski space, thus the CTI curve in this case in R_1^2 index one space curve.

3. An application to the symphony of Beethoven (Vierte Symphony, Op.60)

This section summarizes the statistical components of the aforementioned analysis. There is a clear association between melodic and harmonic weights and time. We give statistical evidence from "Vierte Symphony, Op60", [4], as measured by coding that the pitch of sound, duration of the sound and the sequential cumulative time estimated curve is also given. In order to decide which components are significant or not (i.e., which explanatory variables contribute significantly to the tempo curve), stepwise forward selection is carried out with F test at the level of significance 0.01 as in [5,6]. For the individual curves, a separate stepwise regression is carried out for each individual. The statistics software SPSS was used for the calculations. First we get data for the notes of Vierte Symphony; described in the modeling section. For example; the seventh note of Vierte Symphony corresponds to the point (1, 8, 40, 1) and the eigth note corresponds to the point (4, 4, 48, 1) In this way we have got 64065 points in R^4 that associated to the notes of the symphony. The regression curve of these data is sketched in Figure 1.

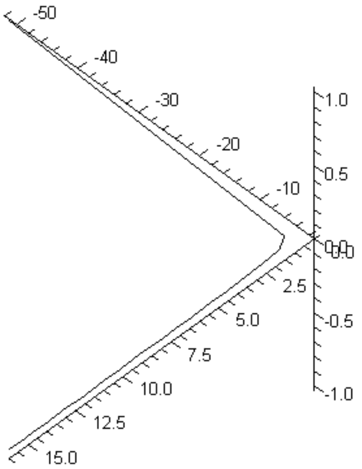

Figure 1.

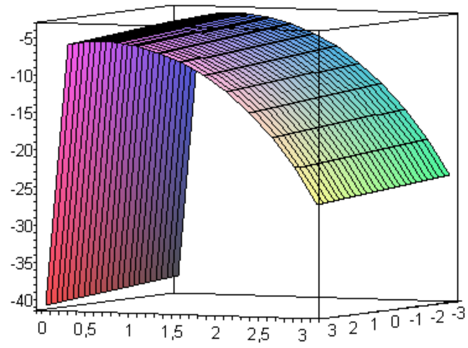

Figure 2.

Using stepwise regression analysis by the program SPSS to data, the following equation is obtained representing the data from Table 3:

$$C(t) = x_2 = -0.12\ln(t) - 1.539\arctan(t) + 0.556\exp(-t) - 2.084(1/t) \quad . \tag{1}$$

This model fits the curvature of the data, in which all coefficients are found meaningful since all of their t values are greater than 2, expect the component of the function cos. See Table 3.

Table 3. Regression Coefficients (Dependent Variable: *V1*)

Model	Unstandardized Coefficients		Standardized Coefficients		
	B	Std Error	Beta	T	P
Constant	11207.977	1854.023		6.1	0.0
V11	-1.766	0.059	-0.120	-29.9	0.0
V20	-7117.103	1180.261	-0.1539	-6.0	0.0
V12	6123.357	809.340	0.556	7.6	0.0
V13	-7871.043	1222280	-2.084	-6.4	0.0

In Table 3, $V1$:Note (pitch of sound), $V2$:duration time *(d)*, $t = V3$: time*(t)*, and other variables described as the following $V4 = d2$, $V5 = d3$, $V6 = e^{-d}$, $V7 = e^{d}$, $V8 = \ln(d)$,

$V9=\sin(d)$, $V10=\cos(d)$, $V11=\ln(t)$, $V12=e^{-t}$, $V13=1/t$, $V14=\sin(t)$, $V15=\sin(3t)$, $V16=\sin(2t)$, $V17=\cos(t)$, $V18=\cos(2t)$, $V19=\cos(3t)$, $V20=\arctan(t)$, $V21=\arctan(d)$

The ANOVA table (see table 4) of the data as follows,

Table 4. Anova Table (Dependent Variable: $V1$)

Model	Sum of Squares	Df	Mean Square	F	p
Regression	237909.3	4	59477.323	231.037	0.000
Residual	16491364.0	64060	257.436		
Total	16729273.0	64064			

We reject the all coefficients zero hypothesis since $p=0$ and because of the p value in the table we can say that all coefficients are computed to an accuracy of 10^{-3}. From the equation (2.1) we parameterized the CDI curve that is the fitting curve of the data as $t \to \left(t, -0.12\ln(t) - 1.539\arctan(t) + 0.556e^{-t} - 2.084\frac{1}{t}, 1\right)$ and the graph is shown in Figure 1.

Using the same data, taking notes *V1* as the dependent variable and $t=V3$ as time, $d=V2$ as time of duration are the independent variables, we apply regression analysis to the data. So, we get the following regression coefficients table, Table.5. We get the parameterized surface, fitting the data in E^3, (in fact it is in E^4 since we use the constant fourth component) as the follows

$$x_2 = -0.247x_1 + 0.1\ln(x_1) - 3.058\arctan(x_3) - 0.409x_3^3 + 1.861\ln(x_3) - 0.69x_3^2 - 1.479e^{-x_3} - 0.17e^{x_3}$$

For the surface ANOVA table as follows;

Table 5. Regression Coefficients for the fitting surface (Dependent Variable: *V1*)

	Unstandardized Coefficients		Standardized Coefficients		
Model	B	Std.Err.	Beta	*t*	P
Constant	235.817	15.985		14.752	0.00
V3	$-3.5\text{x } 10^{-5}$	0	-0.247	-33.090	0.00
V11	1.471	0.11	0.100	13.415	0.00
V21	-212.718	14.16	-3.058	-15.022	0.00
V5	0.001	0	0.409	13.864	0.00
V8	34.122	2.256	1.861	15.127	0.00
V4	-0.053	0.004	-0.690	-14.967	0.00
V6	-191.920	13.250	-1.497	-14.484	0.00
V7	$-5.7\text{x}10^{-0.35}$	0	-0.170	-3.941	0.00

Table 6. ANOVA

Model	Sum of Squares	Df	Mean Square	F	p
Regression	552126.6	8	69015.81	273.279	0.00
Residual	16177146.0	64056	252		
Total	16729273.0	64064			

The surface has the Gaussian curvature $K = 0$, and the surface is not minimal. The surface is plotted as in Figure.2.

4. Comparing with Vierte Symphony and Fur Elis

When we compare Vierte Symphony with Fur Elise: Vierte Symphony has regression curve and regression surface but Fur Elise has only regression curve by stepwise regression method. There are some similarities as they have similar regression graphics. In Table 7, *V20* and *V13* variable predictions are meaningful

Table 7. Regression coefficients (Dependent Variable: *V1*) for Fur Elise

	Unstandardized Coefficients	.	Standardized Coefficients		
Model	B	Std.Err.	Beta	*t*	p
Constant	5540.266	2532.659		2.188	0.00
V20	-3521.974	1612.380	-4.201	-2.184	0.00
V13	-3376.629	1573.637	-4.127	-2.146	0.00

Regression equation is obtained as follows,

Table 8. Anova Table for Fur Elise

Model	Sum of Squares	Df	Mean Square	F	P
Regression	1077.468	2	538.734	5.668	0.004
Residual	113671.000	1196	95.043		
Total	114758.400	1198			

$$x_2 = -4.201 \arctan(x_1) - \frac{4.127}{x_1}$$

Example: "The three piano trios" that Beethoven published in 1795 as his Opus 1 were first performed in 1795. We take a part of Opus 1 [4] and modeled in the four dimensional Minkowski space by using stepwise regression analysis, we find out that the part of Opus 1 corresponds to a hypersurface which has the following equation $x_2 = -0.892x_2^2 - 0.128x_4 - 0.119x_3 - 0.06\log x_3 - 0.06\sin x_1$ that represents the points $(x_1, x_2, x_3, x_4) \in R^4$ corresponding to the data of the "The three piano trios" described as in the modeling paragraph. The following surfaces are intersections of the hypersurface and the coordinate hyperplane. In the case of the hyperplane $z = 1$, the sectional surface has the equation $x_2 = -0.892x_2 - 0.128x_4 - 0.119x_3 - 0.06\sin x_1$.

Similarly in the case of the hyperplane $x = 0$, sectional surface has the equation $x_2 = -0.892x_2 - 0.128x_4 - 0.119x_3 - 0.06\log x_3$ and the graph in Fig.3. The following surfaces are intersections of the hypersurface and the coordinate hyperplanes. In the case of hyperplane $y = 0$, sectional surface has the equation $0 = -0.128x_4 - 0.119x_3 - 0.06\log x_3 - 0.06\sin x_1$.

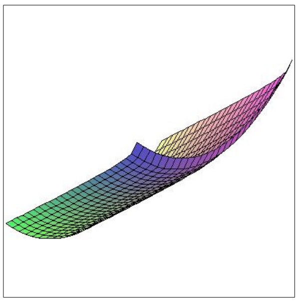

Figure 3. *x=0*, sectional surface

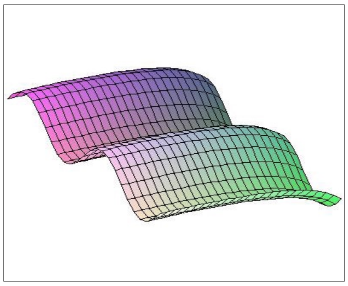

Figure 4. *y=0*, sectional surface

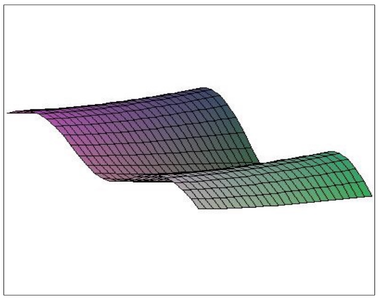

Figure 5. *w=0*, sectional surface

Intersections of the hypersurface and the coordinate hyperplanes plotted in Figure.3, Figure.4, Figure.5, respectively.

For hyperplane $w=0$, sectional surface has the equation $-0.892x_2 - 0.119x_3 - 0.06\log x_3 - 0.06\sin x_1 = 0$.

5. Conclusion

We have shown that a musical composition corresponds to a manifold. We have given a model to find the equation of the manifold in a Lorentzian space. In a special case, we have shown that corresponding manifold can be regarded as a hypersurface in the Minkowski space. On the whole, we think that all the musical compositions have their associated manifold .We have just given two examples from Beethoven in this paper, which show that we can find out the curvature properties of that manifold as if it is in a Lorentzian space.

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