BALKANJM 01 (2013) 1-15

Contents lists available at BALKANJM

BALKAN JOURNAL OF MATHEMATICS

journal homepage: www.balkanjm.com

A Hierarchical Supply Chain Planning Model for a Perishable Item with Stochastic Lifetime

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ARTICLE INFO

Article history:
Received 16 December 2012
Accepted 16 January 2013
Available online 6 February 2013

Keywords:
Stochastic mathematical programming with equilibrium constraints
Bilevel programming
Stochastic programming
Supply chain management
Transportation problem
Perishables
Stochastic lifetime

ABSTRACT

This paper focuses on distribution of a perishable product in a hierarchical supply chain. Due to changeable conditions and the perishable feature of the product, lifetime can be characterized as stochastic. A bilevel programming model is proposed to coordinate the supplier and the retailer. By replacing the second level problem with its KKT optimality conditions, an equivalent single level program of the bilevel programming problem is obtained. A small numerical example is given to illustrate our model.

1. Introduction and literature review

Supply chain is a system consisting of suppliers, manufacturers, distributors, retailers, and transporters who act in a coordinated manner to accomplish product development, marketing, distribution and warehousing tasks and wish to provide a competitive advantage. Supply chain management related problems involve uncertainty due to unexpected failures and fluctuations in parameters. The impact of volatility can be captured by using probability distribution functions that are statistically calculated from historical data. Stochastic

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programming is a technique based on statistical decision theory and concerned with computationally difficult problems under uncertainty.

A government in a distribution network, a service provider in a communication market or a supplier in a supply chain acts as a leader and makes his decision first. As followers, users of those networks, competitors or retailers use that decision as an input to form their strategies. Multi-level programming and game theory address that kind of hierarchical decision making problems and try to find Stackelberg (equilibrium) solutions.

According to Patriksson and Wynter [21], there exists uncertainties in almost all applications of hierarchical problems and neglecting to take into account or simplifying that uncertainty can result costly. Sakawa and Katagiri [25] dealt with bilevel linear programming problems involving random variable coefficients by using chance constrained programming and interactive fuzzy programming. Cromvik and Patriksson [6] gave interesting applications of hierarchical optimization problems with uncertain data. Özaltın et al. [20] introduced a stochastic extension of the bilevel knapsack problem where the leader's 0-1 decisions cause uncertainty on the follower's knapsack capacity considering only a finite set of scenarios. de Kok and Muratore [7] modeled the problem of coordinating and optimizing a supply chain for items with stochastic demand as a bilevel program assuming that each player will implement a just in time policy to minimize their total costs. Ryu et al. [24] addressed a bilevel decision-making problem under uncertainty in which the first level decision maker manages distributions, and the second level decision maker is responsible for production in a supply chain. They presented a solution method based on parametric programming. Roghanian et al. [23] discussed the same problem, but they tackled uncertainty by using chance constrained programming, where constraints may not be satisfied at most some probability. Kalashnikov et al. [12] presented a bilevel multi-stage stochastic optimization model development to balance fuel volumes over a distribution network of natural gas supply chain with random unit prices and demands. In their model, the natural gas shipping company is considered as the leader, and the pipeline operating company is considered as the follower.

Cheng et al. [4] presented a bilevel pricing and ordering model between the manufacturer and the retailer for a uniformly distributed product demand and considered CVaR (conditional or average value-at-risk) measure as the retailer’s objective. Revenue management has become a great tool to obtain optimal price for perishable items with limited capacity in several industries including airlines, hotels, car rentals, and concert organizations. A comprehensive pricing model must involve stochastic, dynamic and game theoretic elements [5]. Two-level stochastic pricing problems can be also found in [2, 13].

In this article, we deal with distribution of a perishable product in a hierarchical supply chain under stochastic environment. Some typical examples of short sales cycle perishables are fresh vegetables or exotic fruits, bread, milk or dairy products, fresh flowers, fresh fish or seafood, meat and frozen foods. By definition, perishables deteriorate and become unusable over a period of time or if exposed to heat, humid or cold air. Consumers prefer long remaining shelf-life products with high quality and taste in every season. The quality of a perishable product decreases rapidly once it is produced and keeps decaying during storage and transportation [3]. Quality loss is a function of both time and temperature. Abuse can result from temperatures in both extremes. For instance, high temperatures cause microbial growth; low temperatures can cause freezing injury, such as discoloration, pitting and off-flavors. More expensive and consuming temperature-controlled vehicles are designed to keep perishable products as fresh as possible during the delivery processes. The producer's benefit increases by using of refrigeration to maintain quality and extend shelf-life. But, high seasonality increases overall costs of transportation [1]. Due to changeable conditions and the perishable feature of the product, lifetime can be characterized as uncertain [8].

The literature regarding two stage or two level supply chain of perishable product problems involving uncertain (random or fuzzy) parameters can be roughly classified into two groups; (i) the newsboy or newsvendor models and extensions: optimal pricing and ordering with fixed or random lifetime perishable product, and with known or uncertain demand, and inventory problems with or without return (buy-back) or revenue-sharing policies [9-11, 14, 16, 18, 19, 22, 26], (ii) production and distribution scheduling, vehicle routing problems with or without time-window constraints [3, 15].

2. Hierarchical programming problems

Bilevel programming (BP) (leader-follower) problems are often considered as hierarchical models or Stackelberg games, in which one player (the leader) has the privilege to play first and announces his decision before the other player (the follower). In BP, the set of decision variables is partitioned between two vectors $\mathbf{x}$ and $\mathbf{y}$. The first level decision maker (the leader) controls over the vector $\mathbf{x} \in \mathbb{R}^m$, and the second level decision maker (the follower) controls over the vector $\mathbf{y} \in \mathbb{R}^n$. The BP problem can be formulated as:

$$\begin{aligned}
&\min_{\mathbf{x},\mathbf{y}} F(\mathbf{x},\mathbf{y}) \\
&\text{subject to } G(\mathbf{x},\mathbf{y}) \le 0 \\
&\qquad \text{where } \mathbf{y} \text{ solves} \\
&\qquad \begin{cases} \min_{\mathbf{y}} f(\mathbf{x},\mathbf{y}) \\ \text{subject to } g(\mathbf{x},\mathbf{y}) \le 0 \end{cases}
\end{aligned} \tag{1}$$

where upper-level variables $\mathbf{x} \in \mathbb{R}^m$, lower-level variables $\mathbf{y} \in \mathbb{R}^n$, upper-level objective function $F : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$, lower-level objective function $f : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$, for upper-level constraints the vector-valued function $G : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^r$ and for lower-level constraints the vector-valued function $g : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^s$.

By replacing the second level problem with its KKT optimality conditions, the equivalent single level program of the BP problem (1) follows as:

$$\begin{aligned}
&\min_{\mathbf{x},\mathbf{y},\lambda} F(\mathbf{x},\mathbf{y}) \\
&\text{subject to } G(\mathbf{x},\mathbf{y}) \le 0 \\
&\qquad g(\mathbf{x},\mathbf{y}) \le 0 \\
&\qquad \nabla_{\mathbf{y}} f(\mathbf{x},\mathbf{y}) + \lambda^T \nabla_{\mathbf{y}} g(\mathbf{x},\mathbf{y}) = 0 \\
&\qquad \lambda_i g_i(\mathbf{x},\mathbf{y}) = 0, \qquad i = 1,\dots,s \\
&\qquad \lambda_i \ge 0, \qquad\qquad i = 1,\dots,s
\end{aligned}$$

where $\lambda \in \mathbb{R}^s$ is the vector of Lagrange multipliers.

A generalization of the BP problem is called Mathematical Programming with Equilibrium Constraints (MPEC) problem that includes first order optimality conditions of another programming problem in its constraints. Stochastic MPEC is defined as a stochastic extension of the MPEC. Two types of formulations have been considered for Stochastic MPEC problems [17]. In *lower-level wait-and-see* formulation, the leader chooses his decision without knowing which way the random event is going to result and the follower made his decision after the random event ω is observed:

$$\mathbf{x} \to \xi(\omega) \to \mathbf{y}(\mathbf{x},\omega)$$

In *here-and-now* formulation, the leader and the follower decide before the random event ω is observed:

$$\mathbf{x} \to \mathbf{y}(\mathbf{x}) \to \xi(\omega) \to \mathbf{z}(\omega)$$

by introducing the recourse variables $\mathbf{z}$.

3. Assumptions, notations and model formulation

In our model, single perishable product subject to continuous decay (not constant shelf-life) is considered. Given customer demands, we assume that lifetime or expiration age of the product is a continuous stochastic variable with known distribution function.

Suppose, there is a leader-follower relationship between the supplier (manufacturer) and the retailer, also they have an agreement to fulfill their customers' demands according to known demands and prices beforehand. We suppose that harvesting or production sites are governed by the leader and distribution centers are operated by the follower. We assume that as the leader, supplier or manufacturer first determines how much perishables he produces or supplies, and then dispatches them to retailer's distribution centers. The leader chooses his quantities to minimize his transportation cost while satisfying capacity and demand requirements. Afterwards, the follower (retailer) determines his quantities send to customers from his distribution centers by considering leader's decision, capacity and demand requirements.

It is also important to transport the perishable product as fresh as possible for the follower. Customers discard spoiled product, so the follower has to bear perishing cost which includes waste disposal cost and replenishment cost in customer zones. Shortage and backlogging are allowed, but penalized since customer demand cannot be met if the product perishes. Inventory holding and oversupply are not allowed due to perishable nature of the product. Travel times between supply chain echelons and reloading times are known in advance. While the follower's objective is to minimize the sum of corresponding total transportation cost and total expected perishing cost, the leader's objective is only to minimize the sum of corresponding total transportation cost.

The notation used in our model is as follows:

3.1. Indices

i, index of production sites, where $i = 1,\ldots,I$, I is the number of production sites;

j, index of distribution centers, where $j = 1,\ldots,J$, J is the number of distribution centers;

k, index of customers, where $k = 1,\ldots,K$, K is the number of customers.

3.2. Capacity and demand parameters

a_i, production or harvesting capacity of production site i;

b_j, capacity of distribution center j;

d_k, demand of customer k.

3.3. Cost parameters

c_{ij}, unit transportation cost from production site i to distribution center j;

e_{jk}, unit transportation cost from distribution center j to customer k;

PC_k, unit perishing (outdate) cost of the product at customer zone k.

3.4. Time parameters

T, random lifetime of the product with distribution function $\Phi_{\bullet}(.)$;

t_{ij}, age of the product transported from production site i to distribution center j;

τ_j, unloading and reloading (processing) time in distribution center j;

T_{jk}, deterministic transportation time for route (j,k)

$\theta_{ijk} = t_{ij} + T_{jk} + \tau_j$, total time the product spends on route $((i,j),k)$.

3.5. Decision variables

$x_{ij} \geq 0$, decision variables of the leader, amount of the product that is to be produced and delivered from production site i to distribution center j;

$y_{ijk} \geq 0$, decision variables of the follower, amount of the product that is to be delivered to customer k through route (i,j).

3.6. The objective function of the follower

As a risk neutral decision maker, the follower's objective is to minimize the sum of corresponding total transportation costs;

$$\sum_{j=1}^{J}\sum_{k=1}^{K} e_{jk} \sum_{i=1}^{I} y_{ijk}$$

and total expected perishing costs;

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} PC_k \ \text{Prob}\{T \le \theta_{ijk}(y_{ijk})\}\, y_{ijk} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} PC_k \ \Phi_{ijk}(\theta_{ijk})\, y_{ijk}$$

such that

$$\theta_{ijk}(y_{ijk}) = \begin{cases} \theta_{ijk}, & y_{ijk} > 0 \\ 0, & y_{ijk} = 0 \end{cases}$$

which generates a non-zero value if there exists a shipment on route $((i, j), k)$. If random lifetime of the product is less than or equal to the total time that the product spends in the system θ_{ijk}, then the product perishes.

3.7. The *here-and-now* formulation

The *here-and-now* formulation suits our case, since the leader and the follower should decide before the product perishes. In order to obtain the Stackelberg solution, the following model must be solved:

$$\begin{aligned}
&\min_{x_{ij}, y_{ijk}} \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij} x_{ij} \\
&\text{subject to } \sum_{j=1}^{J} x_{ij} \le a_i \quad i = 1,\ldots,I \\
&\qquad \sum_{i=1}^{I} x_{ij} \le b_j \quad j = 1,\ldots,J \\
&\qquad x_{ij} \ge 0 \qquad i = 1,\ldots,I;\, j = 1,\ldots,J \\
&\qquad \min_{y_{ijk}} \sum_{j=1}^{J}\sum_{k=1}^{K} e_{jk} \sum_{i=1}^{I} y_{ijk} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} PC_k \ \Phi_{ijk}(\theta_{ijk})\, y_{ijk} \\
&\qquad \text{subject to } \sum_{k=1}^{K} y_{ijk} = x_{ij} \qquad i = 1,\ldots,I;\, j = 1,\ldots,J \\
&\qquad\qquad \sum_{i=1}^{I}\sum_{j=1}^{J} y_{ijk} = d_k \quad k = 1,\ldots,K \\
&\qquad\qquad y_{ijk} \ge 0 \qquad i = 1,\ldots,I;\, j = 1,\ldots,J;\, k = 1,\ldots,K
\end{aligned}$$

where

$$L(\mathbf{y},\mathbf{u},\mathbf{v},\boldsymbol{\omega}) = \sum_{j=1}^{J}\sum_{k=1}^{K} e_{jk} \sum_{i=1}^{I} y_{ijk} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} PC_k \ \Phi_{ijk}(\theta_{ijk})\, y_{ijk} + \sum_{k=1}^{K} u_k (\sum_{i=1}^{I}\sum_{j=1}^{J} y_{ijk} - d_k \ )$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{J} v_{ij} (\sum_{k=1}^{K} y_{ijk} - x_{ij}) - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} \omega_{ijk} y_{ijk}$$

is the Lagrange function for the second level and the variables u_k, v_{ij}, ω_{ijk} are the Lagrange multipliers for $i = 1,\ldots, I; j = 1,\ldots, J; k = 1,\ldots, K$. When the variables ω_{ijk} are vanished, then KKT conditions of the second level problem become:

$$e_{jk} + PC_k \ \Phi_{ijk}(\theta_{ijk}) + u_k + v_{ij} \geq 0, \qquad \text{for } i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$
$$y_{ijk}(e_{jk} + PC_k \Phi_{ijk}(\theta_{ijk}) + u_k + v_{ij}) = 0, \qquad \text{for } i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$
$$\sum_{k=1}^{K} y_{ijk} = x_{ij}, \qquad \text{for } i = 1,\ldots,I; j = 1,\ldots,J$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J} y_{ijk} = d_k, \qquad \text{for } k = 1,\ldots,K$$
$$y_{ijk} \geq 0. \qquad \text{for } i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$

Thus, the equivalent single level programming problem is derived as:

$$\min_{x_{ij}, y_{ijk}} \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij} x_{ij}$$
$$\text{s.t.} \quad \sum_{j=1}^{J} x_{ij} \leq a_i \quad i = 1,\ldots,I$$
$$\sum_{i=1}^{I} x_{ij} \leq b_j \quad j = 1,\ldots,J$$
$$x_{ij} \geq 0 \qquad i = 1,\ldots,I; j = 1,\ldots,J$$
$$e_{jk} + PC_k \ \Phi_{ijk}(\theta_{ijk}) + u_k + v_{ij} \geq 0 \qquad i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$
$$y_{ijk}(e_{jk} + PC_k \Phi_{ijk}(\theta_{ijk}) + u_k + v_{ij}) = 0 \quad i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$
$$\sum_{k=1}^{K} y_{ijk} = x_{ij} \qquad i = 1,\ldots,I; j = 1,\ldots,J$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J} y_{ijk} = d_k \qquad k = 1,\ldots,K$$
$$y_{ijk} \geq 0 \qquad i = 1,\ldots,I; j = 1,\ldots,J; k = 1,\ldots,K$$

When the leader first announces his decisions x_{ij}'s, then decision variable vector of the leader is become known. As a follower, the retailer takes the supplier's optimal decision as an input to establish his best response. Every combination of the follower's reaction is alternative optimal for the leader, since the leader's objective function does not contain the variables y_{ijk}.

Assuming that x_{ij}'s are fixed, the KKT conditions of the second level problem are the same as in the classical transportation problem from $I \times J$ sources (supply points) to K customers with cost minimization. It can be solved with transportation simplex method. But, unit transportation costs should be revised as:

$$Cost((i,j),k) = e_{jk} + PC_k \ \Phi_{ijk}(\theta_{ijk}).$$

4. Proposed solution method

Step 1. Solve leader's problem on relaxed feasible region (minimize the leader's objective function subject to the first and the second level constraints). The optimal solution is (x_{ij}^*, y_{ijk}^*)

Step 2. Calculate each $Cost((i,j),k) = e_{jk} + PC_k \ \Phi_{ijk}(\theta_{ijk})$.

Step 3. Solve the follower's transportation problem:

$$\begin{aligned} &\min_{y_{ijk}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} Cost((i,j),k)\, y_{ijk} \\ &\text{subject to} \quad \sum_{k=1}^{K} y_{ijk} = x_{ij}^* \qquad i=1,\ldots,I;\ j=1,\ldots,J \\ &\qquad\qquad\quad \sum_{i=1}^{I}\sum_{j=1}^{J} y_{ijk} = d_k \qquad k=1,\ldots,K \\ &\qquad\qquad\quad y_{ijk} \geq 0 \qquad i=1,\ldots,I;\ j=1,\ldots,J;\ k=1,\ldots,K \end{aligned}$$

by using transportation simplex method. Update y_{ijk}^*.

Simplex multipliers u_k^* can be calculated by using the optimal tableau for the follower's transportation problem and simplex multipliers v_{ij}^* can be obtained by using

$$v_{ij}^* = \left|\max\{-(Cost((i,j),k) + u_k^*)\}\right|.$$

Equilibrium solution is $\left(x_{ij}^*, y_{ijk}^*\right)$ with simplex multipliers $\left(u_k^*, v_{ij}^*\right)$.

5. Numerical example

There are six production sites, six distribution centers and six customers. Capacities of production sites and distributions centers are given in the following vectors;

$$\mathbf{a} = (\begin{matrix} 200 & 100 & 140 & 160 & 200 & 60 \end{matrix})^T$$

and

$$\mathbf{b} = (\begin{matrix} 200 & 135 & 120 & 140 & 100 & 200 \end{matrix})^T$$

respectively. Customer demands are $\mathbf{d} = (80 \quad 60 \quad 200 \quad 90 \quad 200 \quad 100)^T$.

Cost parameters are given in the following matrices;

$$\mathbf{PC} = (400 \quad 500 \quad 480 \quad 520 \quad 560 \quad 440)^T, \ \mathbf{c} = \begin{pmatrix} 31.0 & 21.0 & 18.0 & 21.5 & 36.5 & 31.5 \\ 48.5 & 37.5 & 36.0 & 40.0 & 55.0 & 46.5 \\ 45.0 & 33.5 & 33.0 & 39.0 & 54.0 & 48.0 \\ 55.5 & 44.0 & 44.0 & 49.5 & 65.0 & 57.0 \\ 41.5 & 30.0 & 38.0 & 43.0 & 56.0 & 55.5 \\ 28.5 & 18.5 & 24.0 & 36.5 & 46.0 & 51.0 \end{pmatrix},$$

and

$$\mathbf{e} = \begin{pmatrix} 43.0 & 52.5 & 62.0 & 60.5 & 50.0 & 54.5 \\ 32.5 & 43.5 & 54.0 & 56.0 & 48.0 & 55.0 \\ 30.0 & 40.0 & 50.0 & 49.0 & 39.0 & 45.0 \\ 23.0 & 33.0 & 43.0 & 43.5 & 35.0 & 41.5 \\ 12.0 & 22.5 & 34.0 & 38.0 & 33.0 & 41.0 \\ 23.0 & 33.0 & 44.5 & 51.5 & 48.0 & 56.5 \end{pmatrix}.$$

Time parameters expressed in the same units are given in the following matrices;

$$\tau = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}^T,$$

$$\mathbf{t} = \begin{pmatrix} 2.5 & 1.5 & 5.0 & 2.0 & 5.0 & 5.5 \\ 4.0 & 4.0 & 1.5 & 4.5 & 1.0 & 3.5 \\ 3.0 & 1.0 & 4.5 & 2.5 & 3.5 & 3.5 \\ 3.5 & 4.5 & 1.0 & 4.5 & 3.0 & 3.0 \\ 1.5 & 2.0 & 4.5 & 1.5 & 3.5 & 3.5 \\ 2.5 & 3.5 & 3.0 & 3.0 & 1.0 & 0.5 \end{pmatrix}, \text{ and } \mathbf{T} = \begin{pmatrix} 3.5 & 0.5 & 1.0 & 4.5 & 3.5 & 1.5 \\ 3.5 & 3.5 & 3.5 & 3.5 & 0.5 & 3.5 \\ 1.0 & 3.5 & 3.5 & 2.0 & 2.5 & 4.0 \\ 4.0 & 1.0 & 0.5 & 4.5 & 4.0 & 1.5 \\ 4.0 & 4.0 & 4.0 & 3.0 & 1.0 & 4.0 \\ 3.5 & 3.0 & 3.0 & 3.5 & 2.5 & 3.0 \end{pmatrix}.$$

Optimal values of leader's variables are given in (Table 1). We use three type of distribution function, first uniform distribution, i.e., the lifetime of the product has uniform distribution on the interval [6,10] with mean 8 units of time, second piecewise-uniform distribution:

$$\Phi(t) = \begin{cases} 0, & t < 6 \\ 0.4(t-6), & 6 \le t < 7 \\ 0.4 + 0.3(t-7), & 7 \le t < 8 \\ 0.7 + 0.2(t-8), & 8 \le t < 9 \\ 0.9 + 0.1(t-9), & 9 \le t < 10 \\ 1, & 10 \le t \end{cases}$$

with mean 7.5 units of time, and, third exponential distribution with $\lambda = 0.125$ or mean 8 units of time.

Table 1. Optimal solutions for the leader's problem on relaxed feasible region.

	DC1	DC2	DC3	DC4	DC5	DC6
P1	0	0	32.5	140	0	27.5
P2	0	0	0	0	0	100
P3	52.5	0	87.5	0	0	0
P4	22.5	0	0	0	0	7.5
P5	65	135	0	0	0	0
P6	60	0	0	0	0	0

In Step 2, probable total times on routes, distribution function values on those probable routes, and $Cost((i, j), k)$ values can be obtained easily by using a simple computer code. For piecewise-uniform distribution, a simple nonlinear optimization model is solved to obtain the multipliers: $\lambda_{ijk}^r \ge 0, \quad r = 1, \dots, 7$ from the following equations:

$$\sum_{r=1}^{7} \lambda_{ijk}^{r} = 1,$$

$$t_{ij} + \tau_j + T_{jk} = 0\lambda_{ijk}^1 + 6\lambda_{ijk}^2 + 7\lambda_{ijk}^3 + 8\lambda_{ijk}^4 + 9\lambda_{ijk}^5 + 10\lambda_{ijk}^6 + 12\lambda_{ijk}^7,$$

$$\Phi(t_{ij} + \tau_j + T_{jk}) = 0.4\lambda_{ijk}^3 + 0.7\lambda_{ijk}^4 + 0.9\lambda_{ijk}^5 + 1\lambda_{ijk}^6 + 1\lambda_{ijk}^7.$$

Non-zero optimal solutions for respective distribution functions are given in (Table 2), (Table 3), and (Table 4).

Table 2. Optimal solutions for the follower's problem, uniform distribution.

P1.DC3.C1	32.5	P1.DC4.C2	60	P1.DC4.C3	80	P1.DC6.C6	27.5
P2.DC6.C5	100	P3.DC1.C3	52.5	P3.DC3.C1	47.5	P3.DC3.C4	40
P4.DC1.C3	22.5	P4.DC6.C5	7.5	P5.DC1.C3	45	P5.DC1.C5	7.5
P5.DC1.C6	12.5	P5.DC2.C4	50	P5.DC2.C5	85	P6.DC1.C6	60

Table 3. Optimal solutions for the follower's problem, piecewise-uniform distribution.

P1.DC3.C4	32.5	P1.DC4.C2	60	P1.DC4.C3	80	P1.DC6.C4	7.5
P1.DC6.C6	20	P2.DC6.C5	100	P3.DC1.C3	52.5	P3.DC3.C1	80
P3.DC3.C4	7.5	P4.DC1.C3	22.5	P4.DC6.C5	7.5	P5.DC1.C3	45
P5.DC1.C6	20	P5.DC2.C4	42.5	P5.DC2.C5	92.5	P6.DC1.C6	60

Table 4. Optimal solutions for the follower's problem, exponential distribution.

P1.DC3.C1	32.5	P1.DC4.C3	140	P1.DC6.C1	7.5	P1.DC6.C6	20
P2.DC6.C4	42.5	P2.DC6.C5	57.5	P3.DC1.C6	52.5	P3.DC3.C1	40
P3.DC3.C4	47.5	P4.DC1.C6	22.5	P4.DC6.C5	7.5	P5.DC1.C2	60
P5.DC1.C3	5	P5.DC2.C5	135	P6.DC1.C3	55	P6.DC1.C6	5

Table 5. Simplex multipliers, uniform distribution.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
u_k	-130	-223	-233	-294	-221	-225.5
v_{ij}	**DC1**	**DC2**	**DC3**	**DC4**	**DC5**	**DC6**
P1	171	238	0	190	48	216
P2	171	173	245	190	256	33
P3	171	238	50	190	188	33
P4	171	173	245	190	188	103
P5	171	173	50	190	188	33
P6	171	173	245	190	256	242,5

6. Conclusion

This paper focuses on determining the Stackelberg solution of two-level hierarchical transportation problem considering perishable product characteristics. The proposed model is constructed using SMPEC concepts. The model is applicable when there is a hierarchically structured cold chain. We examine three kinds of distribution functions with close mean values. The results showed that equilibrium solution is very sensitive to shape of the distribution function shape. In the future study, stochastic demand (or other coefficients) case, indivisible product case (with integer programming), vehicle routing model, more echelons or any combinations of these four cases may be considered.

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