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On Solving Non Linear Complex Partial Derivate Equations by Using Differential Transform Method

Murat Duz[*a] and Ugur Ilter [a]

[a] *Department of Mathematics, Karabuk University, Karabuk, Turkey*

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ABSTRACT

In this study, first order nonlinear complex equations were solved using two dimensional differential transform method. Firstly we separated real and imaginary parts these equations. Thus two equality was obtained .Later using two dimensional differential transform we obtained real and imaginary parts of solution.

1. Introduction

The concept of differential transform (with one dimension) was first proposed and applied to solve linear and nonlinear initial value problems in electric circuit analysis by Zhou [1] . By using one dimension differential transform method, nonlinear differential equations were solved in [4]. Partial differential equations were solved by using two dimensional differential transform method in [2,3].

The differential transform method has solution in the form of a polynomial. The differential transform is an iterative procedure. This method consist of computing the coefficients of Taylor series of solution by using initial value. In this paper, linear complex partial differential equations were solved by using two dimensional differential transform method. Let $w=w(z,\bar{z})$ be a complex function. Here $z=x+iy, w(z,\bar{z})=u(x,y)+iv(x,y)$. Derivative according to z and $\bar{z}$ of $w=w(z,\bar{z})$ is defined as follows:

[*] Corresponding author: *E-mail*: mduz@karabuk.edu.tr (M. Duz).

$$\frac{\partial w}{\partial z}=\frac{1}{2}\left(\frac{\partial w}{\partial x}-i\frac{\partial w}{\partial y}\right) \tag{1}$$

$$\frac{\partial w}{\partial \overline{z}}=\frac{1}{2}\left(\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y}\right) \tag{2}$$

$$\frac{\partial w}{\partial x}=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x} \tag{3}$$

$$\frac{\partial w}{\partial y}=\frac{\partial u}{\partial y}+i\frac{\partial v}{\partial y} \tag{4}$$

$$\frac{\partial w}{\partial z}=\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{i}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial v}{\partial y}\right) \tag{5}$$

$$\frac{\partial w}{\partial \overline{z}}=\frac{1}{2}\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)+\frac{i}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{6}$$

2. Two Dimensional Differential Transform

Definition 2.1. Two dimensional differential transform of function $f\left(x,y\right)$ is defined as follows

$$F\left(k,h\right)=\frac{1}{k!.h!}\left[\frac{\partial^{k+h} f\left(x,y\right)}{\partial x^k \partial y^h}\right]_{x=0,\,y=0} \tag{7}$$

In Eq. (7), $f\left(x,y\right)$ is original function and $F\left(k,h\right)$ is transformed function, which is called T - function is brief.

Definition 2.2. Differential inverse transform of $F\left(k,h\right)$ is defined as follows

$$f(x,y)=\sum_{(k=0)}^{\infty}\sum_{(h=0)}^{\infty}F(k,h)x^k y^h \tag{8}$$

$$f\left(x,y\right)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!.h!}\left[\frac{\partial^{k+h} f\left(x,y\right)}{\partial x^k \partial y^h}\right]_{x=0,\,y=0} x^k y^h \tag{9}$$

Eq. (9) implies that the concept of two dimensional differential transform is derived from two dimensional Taylor series expansion.

Theorem 2.1. [2,3]: If $w\left(x,y\right)=u\left(x,y\right)\mp v\left(x,y\right)$ then $W\left(k,h\right)=U\left(k,h\right)\mp V\left(k,h\right)$.

Theorem 2.2. [2,3]: If $w\left(x,y\right)=\lambda u\left(x,y\right)$then $W\left(k,h\right)=\lambda U\left(k,h\right)$.

Theorem 2.3. [2,3]: If $w\left(x,y\right)=\frac{\partial u\left(x,y\right)}{\partial x}$ then $W\left(k,h\right)=\left(k+1\right)U\left(k+1,h\right)$.

Theorem 2.4. [2,3]: If $w(x,y)=\dfrac{\partial u(x,y)}{\partial y}$ then $W(k,h)=(h+1)U(k,h+1)$.

Theorem 2.5. [2,3]: If $w(x,y)=\dfrac{\partial^{r+s}u(x,y)}{\partial x^r \partial y^s}$ then

$$W(k,h)=(k+1)(k+2)\ldots(k+r)(h+1)(h+2)\ldots(h+s)U(k+r,h+s)$$

Theorem 2.6. [2,3]: If $w(x,y)=u(x,y).v(x,y)$ then

$$W(k,h)=\sum_{k=0}^{r}\sum_{s=0}^{h}U(r,h-s).V(k-r,s).$$

Theorem 2.7. [2,3]: If $w(x,y)=x^m y^n$ then $W(k,h)=\delta(k-m,h-n)$

3. Using Two-Dimensional Differential Transform to Solve Non Linear Complex Partial Derivative Equations

To demonstrate how to use two-dimensional transform to solve complex partial equations are

solved here.

Example 3.1. Solve the following initial-value problem

$$w\frac{\partial w}{\partial \overline{z}}=z^2+\overline{z} \tag{10}$$

$$w(x,0)=x^2+x \tag{11}$$

Since $w=u+iv$ and from equation (5) we obtain that

$$\frac{u+iv}{2}\left[\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}+i\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]=(x+iy)^2+x-iy \tag{12}$$

Therefore

$$u\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)-v\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=2x^2-2y^2+2x \tag{13}$$

$$v\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)+u\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=4xy-2y \tag{14}$$

Following equalities are found by differential transform of (13) and (14)

$$\begin{aligned}&\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(k-r+1)U(k-r+1,s)-\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)(s+1)U(k-r,s+1)\\&=2\delta(k-2,h)-2\delta(k,h-2)+2\delta(k-1,h)\end{aligned} \tag{15}$$

$$\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)(k-r+1)U(k-r+1,s)-\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)(s+1)V(k-r,s+1)$$
$$-\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(s+1)U(k-r,s+1)=4\delta(k-1,h-1)-2\delta(k,h-1) \quad (16)$$

From the equality (11), we obtain

$$\begin{aligned} &U(0,0)=0 \quad U(2,0)=1 \\ &U(1,0)=1 \quad U(i,0)=0 \quad (i=3,4,5,\dots) \\ &V(i,0)=0 \quad (i=0,1,2,\dots) \end{aligned} \quad (17)$$

When $h=0$ is written in (15) equality, we obtain

$$\sum_{r=0}^{k}U(r,0)(k-r+1)U(k-r+1,0)-\sum_{r=0}^{k}U(r,0)V(k-r,1)-\sum_{r=0}^{k}V(r,0)U(k-r,1)$$
$$-\sum_{r=0}^{k}V(r,0)(k-r+1)V(k-r+1,0) \quad (18)$$
$$=2\delta(k-2,h)+2\delta(k-1,h)$$

Hence, we obtain following equality by using (17) and (18)

$$\sum_{r=0}^{k}U(r,0)(k-r+1)U(k-r+1,0)-\sum_{r=0}^{k}U(r,0)V(k-r,1)$$
$$=2\delta(k-2,h)+2\delta(k-1,h) \quad (19)$$

If we write $k=1,k=2,k=3,k=4$ in (19), then we obtain

$$V(0,1)=-1,V(1,1)=2,V(2,1)=0,V(3,1)=0 \quad (20)$$

respectively. Furthermore it is clear that for $n\geq 2$

$$V(n,1)=0 \quad (21)$$

When $h=0$ is written in equality (16), we obtain

$$\sum_{r=0}^{k}V(r,0)(k-r+1)U(k-r+1,0)-\sum_{r=0}^{k}V(r,0)V(k-r,1)$$
$$+\sum_{r=0}^{k}U(r,0)U(k-r,1)+\sum_{r=0}^{k}U(r,0)(k-r+1)V(k-r+1,0)=0 \quad (22)$$

Hence we obtain following equality by using (17), (22)

$$\sum_{r=0}^{k}U(r,0)U(k-r,1)=0 \quad (23)$$

If we write $k=1, k=2, k=3$ in (23), then we obtain

$$U(0,1)=0, U(1,1)=0, U(2,1)=0 \tag{24}$$

respectively. So it is easy to see that for

$$\forall U(n,1)=0 \tag{25}$$

When $h=1$ is written in equality (15), we obtain

$$\sum_{r=0}^{k} U(r,0)\, 2V(k-r,2)=0 \tag{26}$$

Similarly if we write $k=1, k=2$ in equality (26), then we obtain

$$V(0,2)=V(1,2)=0 \tag{27}$$

respectively. Clearly for

$$\forall V(n,2)=0 \tag{28}$$

When $h=1$ is written in (16)

$$\begin{aligned}
&\sum_{r=0}^{k} V(r,1)(k-r+1)U(k-r+1,0)-\sum_{r=0}^{k} V(r,0)(k-r,1)U(k-r+1,1)\\
&-\sum_{r=0}^{k} V(r,1)V(k-r,1)-\sum_{r=0}^{k} 2V(r,0)V(k-r,2)+\sum_{r=0}^{k} U(r,1)U(k-r,1)\\
&+\sum_{r=0}^{k} 2U(r,0)U(k-r,2)+\sum_{r=0}^{k} U(r,1)(k-r+1)V(k-r+1,0)\\
&+\sum_{r=0}^{k} U(r,0)(k-r+1)V(k-r+1,1)=4\delta(k-1,0)-2\delta(k,0)
\end{aligned} \tag{29}$$

Hence we obtain following equality by using (17), (29)

$$\begin{aligned}
&\sum_{r=0}^{k} V(r,1)(k-r+1)U(k-r+1,0)+\sum_{r=0}^{k} 2U(r,0)U(k-r,2)\\
&-\sum_{r=0}^{k} V(r,1)V(k-r,1)+\sum_{r=0}^{k} U(r,0)(k-r+1)V(k-r+1,1)\\
&=4\delta(k-1,0)-2\delta(k,0)
\end{aligned} \tag{30}$$

When $k=1, k=2, k=3$ are written in equality (30), we obtain

$$\begin{aligned}
&U(0,2)=-1\\
&U(1,2)=0\\
&U(2,2)=0
\end{aligned} \tag{31}$$

respectively. Clearly for $\forall n \in N^{+}$

$$U(n,2)=0 \tag{32}$$

It is obtained that order components of U and V are equal zero by using (15) and (16). Thus, we find that

$$u(x,y)=x^2-y^2+x \tag{33}$$

and

$$v(x,y)=2xy-y \tag{34}$$

From equalities (33) and (34) we get that

$$\begin{aligned} w(x,y)&=u(x,y)+iv(x,y)\\ &=x^2-y^2+x+i(2xy-y)\\ &=\left(\frac{z+\overline{z}}{2}\right)^2-\left(\frac{z-\overline{z}}{2i}\right)^2+\frac{z+\overline{z}}{2}+i\left[2\left(\frac{z+\overline{z}}{2}\right)\left(\frac{z-\overline{z}}{2i}\right)-\left(\frac{z-\overline{z}}{2i}\right)\right]\\ &=z^2+\overline{z}\end{aligned}$$

Example 3.2. Solve the following initial-value problem

$$w\frac{\partial w}{\partial z}+\overline{w}\frac{\partial w}{\partial \overline{z}}=2z+2\overline{z} \tag{35}$$

$$w(x,0)=2x \tag{36}$$

Since $w=u+iv$ and from equation (5) and (6) we obtain that

$$\frac{u+iv}{2}\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}-i\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}\right)+\frac{u-iv}{2}\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}+i\frac{\partial u}{\partial y}-\frac{\partial v}{\partial y}\right)=4x \tag{37}$$

Therefore we get that:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=4x \tag{38}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=0 \tag{39}$$

From differential transform of (38) and (39) equalities

$$\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(k-r+1)U(k-r+1,s)+\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)(s+1)U(k-r,s+1) = 4\delta(k-1,h) \tag{40}$$

$$\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(k-r+1)V(k-r+1,s)-\sum_{r=0}^{k}\sum_{s=0}^{h}V(r,h-s)(s+1)V(k-r,s+1)=0 \tag{41}$$

From (36), we get

$$\begin{aligned} &U(0,0)=0 \quad U(i,0)=0 \quad (i=2,3,4,\dots)\\ &U(1,0)=2\\ &V(i,0)=0 \quad (i=0,1,2,\dots)\end{aligned} \tag{42}$$

It is easy to see that other components U and V are equal zero by using (40), (41), (42). Thus, we find that

$$u(x,y)=2x \tag{43}$$

and

$$v(x,y)=0 \tag{44}$$

From (44) and (45) equalities, we get that

$$\begin{aligned} w(x,y)&=u(x,y)+iv(x,y)\\ &=2x\\ &=z+\overline{z}\end{aligned}$$

Example 3.3. Solve the following initial-value problem

$$\frac{\partial w}{\partial z}.\frac{\partial w}{\partial \overline{z}}=4z \tag{45}$$

$$w(x,0)=x^2+2x \tag{46}$$

Since $w=u+iv$ equation (45) is equivalent to equation that

$$\left[\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}-i\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}\right].\left[\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}+i\frac{\partial u}{\partial y}-\frac{\partial v}{\partial y}\right]=4(x+iy) \tag{47}$$

Equation (48) is equivalent to partial differential equation system that:

$$\frac{\partial u}{\partial x}.\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}.\frac{\partial v}{\partial y}-\frac{\partial v}{\partial x}.\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}.\frac{\partial u}{\partial y}=4x \tag{48}$$

$$2\frac{\partial u}{\partial x}.\frac{\partial v}{\partial x}+2\frac{\partial u}{\partial y}.\frac{\partial v}{\partial y}=4y \tag{49}$$

From the differential transform of (48) and (49) we get that:

$$\begin{aligned}&\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)U(r+1,h-s)(k-r+1)U(k-r+1,s)\\&+\sum_{r=0}^{k}\sum_{s=0}^{h}(h-s+1)U(r,h-s+1)(s+1)U(k-r,s+1)\\&-\sum_{r=0}^{k}\sum_{s=0}^{h}(h-s+1)V(r,h-s+1)(s+1)V(k-r,s+1)\\&-\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)V(r+1,h-s)(k-r+1)V(k-r+1,s)=4\delta(k-1,h)\end{aligned} \tag{50}$$

$$\begin{aligned}&\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)U(r+1,h-s)(k-r+1)V(k-r+1,s)\\&+\sum_{r=0}^{k}\sum_{s=0}^{h}(h-s+1)U(r,h-s+1)(s+1)V(k-r,s+1)=2\delta(k,h-1)\end{aligned} \tag{51}$$

From (47) equality is obtained that:

$$\begin{aligned}&U(0,0)=0 \quad U(2,0)=1 \quad (i=3,4,5,\ldots)\\&U(1,0)=2 \quad U(i,0)=0\\&V(i,0)=0 \quad (i=0,1,2,\ldots)\end{aligned} \tag{52}$$

We get that:

$$\begin{aligned}&U(0,2)=-1,\\&V(0,1)=-2\\&V(1,1)=2\end{aligned} \tag{53}$$

and remaining components of U and V are equal zero.

Hence we obtain

$$u(x,y)=x^2+2x-y^2 \tag{54}$$

and

$$v(x,y)=2xy-2y \tag{55}$$

From (54) and (55) equalities we get that

$$w(x,y) = u(x,y) + iv(x,y)$$

$$= x^2 + 2x - y^2 + i(2xy - 2y)$$

$$= \left(\frac{z+\overline{z}}{2}\right)^2 + z + \overline{z} - \left(\frac{z-\overline{z}}{2i}\right)^2 + 2i\left[\left(\frac{z+\overline{z}}{2}\right)\left(\frac{z-\overline{z}}{2i}\right) - \left(\frac{z-\overline{z}}{2i}\right)\right]$$

$$= z^2 + 2\overline{z}$$

4. Conclusion

In this study nonlinear complex equations can be solved using two dimensional differential transform method. As a future work, we plan to solve higher order linear or nonlinear complex equation, systems of complex equation and fractional order complex equation.

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