Fuzzy TOPSIS Method In Group Decision Making and An Application: Personnel Selection

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ARTICLE INFO

Article history:
Received: January 17, 2015
Accepted: February 15, 2015
Available online: June 8, 2015

Keywords:
Personnel Selection
Decision making
Fuzzy Sets and TOPSIS

ABSTRACT

Personnel selection is a multi-criteria decision problem and has a strategic importance for many companies. In this problem a decision support model is provided in order to select the best alternative among five alternatives. In the decision support model developed, because of the fuzziness of the evaluation processus, the fuzzy TOPSIS method which allows to make decisions using the intervals is applied. At the end of the study, according to the evaluations under the criteria defined, the fourth alternative is found out the best alternative.

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1. Introduction

In the global market, modern organizations face high levels of competition. In the wake of increasingly competitive world market the future survival of most companies, depends mostly on the dedication of their personnel to companies. Employee or personnel performances such as capability, knowledge, skill, and other abilities play an important role in the success of an organization. The main goal of organizations is to seek more powerful ways of ranking of a set employee or personnel who have been evaluated in terms of different competencies. Great deal of attention in literature was given for the selection of eligible and adequate person among alternative rivals and extensively conducted review can be found in [18]. The objective of a selection process depends mainly on assessing the differences among candidates and predicting the future performance. Latter is a challenging task since larger samples are required and other temporal changes may affect employees. Personality factors are generally described as emotional stability, extraversion, openness, agreeableness and conscientiousness [12]. [19] determined seven criteria from overview of job description: written communication, oral communication, planning, organizing ability, team player, decisiveness, and working independently. One of the techniques concerning the selection of

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personnel to fill new positions is to have interviews with related personnel. [18] and [6] present notable ability and availability of interviews to predict the performance of the personnel in the job. The usages of different methods in some European countries are given in [10].

In this paper, the extended TOPSIS method is considered which was originally proposed by [6]. Triangular fuzzy numbers are used to express the linguistic ratings of decision makers who cannot estimate their preferences with an exact numerical value. In Fuzzy TOPSIS the decision makers use the linguistic variables to assess the importance of the criteria and to evaluate the each alternative with respect to each criterion. These linguistic variables are converted into triangular fuzzy numbers and fuzzy decision matrix is formed. Then normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix are formed. After FPIS and FNIS are defined, the closeness coefficient of each alternative is calculated. According to these values, decision maker can determine the order of the alternatives and can choose the best one.

The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology, fuzzy TOPSIS. The application of the proposed framework to personnel selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy Sets and Numbers

"Not very clear", “probably so”, “very likely”… These terms of expression can be heard very often in daily lie and their commonality is that they are more or less tainted with uncertainty. With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision making is not taken into account [20]. In order to deal with vagueness of human thought, [25] first introduced the fuzzy set theory. A fuzzy is an extension of a crisp set. Crisp sets only allow full membership or no membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set.

The classical set theory is built on the fundamental concept of set of which is either a member or not a member. A sharp, crisp and ambiguous distinction exists between a member and non-member for any well-defined set of entities in this theory and there is a very precise and clear boundary to indicate if an entity belongs to the set. But many real-world applications cannot be described and handled by classical set theory [6].

Zadeh proposed to use values ranging from 0 to 1 for showing the membership of the objects in a fuzzy set. Complete non-membership is represented by 0 and complete membership as 1. Values between 0 and 1 represent intermediate degrees of membership. Fuzzy sets and fuzzy logic are powerful mathematic tools for modeling uncertain systems in industry, nature and humanity. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution[5].

A fuzzy number is convex set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1 [11]. It is possible to use different fuzzy numbers according to the situation. Generally in practice, triangular and trapezoidal fuzzy
numbers are used[3]. In applications, it is convenient to work with triangular fuzzy numbers because of their simplicity.

A fuzzy set \( \tilde{A} \) can be defined mathematically by a membership function \( \mu_A(X) \), which assigns each element \( x \) in the universe of discourse \( X \) a real number in the interval [0,1]. A triangular fuzzy number \( \tilde{A} \) can be defined by a triplet \( (a, b, c) \) and the membership function \( \mu_A(X) \) is defined as

\[
\mu_A(X) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{x-a}{b-c}, & b \leq x \leq c, \\
0, & \text{otherwise}
\end{cases}
\] (1)

Basic arithmetic operations on triangular fuzzy numbers \( A_1 = (a_1, b_1, c_1) \) where \( a_1 \leq b_1 \leq c_1 \) and \( A_2 = (a_2, b_2, c_2) \) where \( a_2 \leq b_2 \leq c_2 \) can be shown as follows:

Addition : \( A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \) (2)

Subtraction : \( A_1 - A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2) \) (3)

Multiplication : \( k \otimes A_1 = \begin{cases} 
(ka_1, kb_1, kc_1), & k > 0 \\
(kc_1, kb_1, ka_1), & k < 0
\end{cases} \)

\( A_1 \otimes A_2 = (a_1 a_2, b_1 b_2, c_1 c_2) \) (4)

Division : \( A_1 \div A_2 = \left( \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2} \right) \) (5)

The distance between two triangular fuzzy numbers can be calculated by using vertex method as [8]:

\[
d_v(\tilde{m}, \tilde{n}) = \frac{1}{\sqrt{3}} \left[ (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2 \right]
\] (6)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications. Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation [15]. A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms [26].
3. The Fuzzy TOPSIS Method

TOPSIS views a MADM problem with \( m \) alternatives as a geometric system with \( m \) points in the \( n \)-dimensional space. The method is based on the concept that the chosen alternative should have the shortest distance from the positive-ideal solution and the longest distance from the negative-ideal solution. TOPSIS defines an index called similarity to the positive-ideal solution and the remoteness from the negative-ideal solution. Then the method chooses an alternative with the maximum similarity to the positive-ideal solution [21]. It is often difficult for a decision-maker to assign a precise performance rating to an alternative for the attributes under consideration. This section extends the TOPSIS to the fuzzy environment [24]. This method is particularly suitable for solving the group decision-making problem under fuzzy environment. The mathematics concept borrowed from [2], [6] and [21]:

The fuzzy TOPSIS method can be described by the help of following sets: \( n \) be described by the help of following sets [8]

- a set of \( k \) decision-makers called \( E = \{D_1, D_2, ..., D_k\} \)
- a set of \( m \) possible alternatives called \( A = \{A_1, A_2, ..., A_m\} \)
- a set of \( n \) criteria \( C = \{C_1, C_2, ..., C_n\} \) with which alternative performance are measured.
- a set of performance ratings of \( A_i = \{i = 1, 2, ..., m\} \) with respect to criteria \( C_j = \{j = 1, 2, ..., n\} \) called \( X = \{x_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n\} \)

In a decision committee that has \( K \) decision makers, fuzzy rating of each decision maker \( D_k = (k = 1, 2, ..., K) \) can be represented as a triangular fuzzy number \( \tilde{R}_k = (a_k, b_k, c_k) \); \( (k = 1, 2, ..., K) \). Then the aggregated fuzzy number \( \tilde{R}_k = (k = 1, 2, ..., K) \) with membership function \( \mu_k(X) \).

The steps of the method can be described as following:

**Step 1:** Determine the weighting of evaluation criteria
The importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables (as Table 1) and the linguistic variables for the evaluation of the alternatives by the decision makers are demonstrated in Table 2.

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Corresponding Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very High(VH)</td>
<td>(0.8, 1, 1)</td>
</tr>
<tr>
<td>High(H)</td>
<td>(0.7, 0.8, 0.9)</td>
</tr>
<tr>
<td>Medium High(MH)</td>
<td>(0.5, 0.6, 0.8)</td>
</tr>
<tr>
<td>Medium</td>
<td>(0.4, 0.5, 0.6)</td>
</tr>
<tr>
<td>Medium Low</td>
<td>(0.2, 0.35, 0.5)</td>
</tr>
<tr>
<td>Low</td>
<td>(0.1, 0.2, 0.3)</td>
</tr>
<tr>
<td>Very Low</td>
<td>(0, 0, 0.2)</td>
</tr>
</tbody>
</table>

Table 1 Linguistic Variables Used For The Evaluation Of The Criteria
Step 2: Construct the fuzzy decision matrix and choose the appropriate linguistic variables for the alternatives with respect to criteria.

\[
D = \begin{bmatrix}
A_{11} & A_{12} & \ldots & A_{1n} \\
A_{21} & A_{22} & \ldots & A_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{m1} & A_{m2} & \ldots & A_{mn}
\end{bmatrix}
\]

\[
\tilde{x}_{ij} = \frac{1}{k} \left( \tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \ldots + \tilde{x}_{ij}^k \right)
\]  

where \( \tilde{x}_{ij}^k \) is the rating of alternative \( A_i \) with respect to criterion \( C_j \) evaluated by \( K \) expert and \( \tilde{x}_{ij}^k = (a_j^k, b_j^k, c_j^k) \).

Step 3: Normalize the fuzzy decision matrix

To avoid the complicated normalization formula used in classical TOPSIS, the linear scale transformation can be used to transform the various criteria scales into a comparable scale. Therefore it is possible to obtain the normalized fuzzy decision matrix which can be denoted by \( \tilde{R} \) and it is shown as following formula:

\[
\tilde{R} = \left[ \tilde{r}_{ij} \right]_{m\times n} \quad i=1,2,\ldots,m ; j=1,2,\ldots,n
\]

\[
\tilde{r}_{ij} = \left( \frac{a_j}{c_j^+}, \frac{b_j}{c_j^+}, \frac{c_j}{c_j^+} \right) \quad c_j^+ = \max_i c_{ij}
\]  

The normalized \( \tilde{r}_{ij} \) are still triangular fuzzy numbers. For trapezoidal fuzzy numbers, the normalization process can be conducted in the same way. Considering the different importance of each criterion, the weighted fuzzy normalized decision matrix is shown as following matrix:

\[
\tilde{V} = \left[ \tilde{v}_{ij} \right]_{m\times n}
\]

\[
\tilde{v}_{ij} = \tilde{r}_{ij} \otimes w_j
\]
Step 4: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).

According to the weighted normalized fuzzy decision matrix, we know that the elements $\tilde{v}_{ij}$ are normalized positive TFNs and their ranges belong to the closed interval $[0, 1]$. Then, we can define the FPIS $A^+$ and FNIS $A^-$ as following formula:

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, ..., \tilde{v}_n^+)$$  \hspace{1cm} (11)

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, ..., \tilde{v}_n^-)$$  \hspace{1cm} (12)

where $\tilde{v}_j^+ = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0) \quad j=1,2,...,n$

Step 5: Calculate the distance of each alternative from FPIS and FNIS.

The distances $d_i^+$ and $d_i^-$ of each alternative $A^+$ and $A^-$ can be currently calculated by the area compensation method where $d_p(...)$ is the distance measurement between two fuzzy numbers:

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+) \hspace{1cm} i=1,2,...,m \hspace{0.5cm} j=1,2,...,n$$ \hspace{1cm} (13)

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-) \hspace{1cm} i=1,2,...,m \hspace{0.5cm} j=1,2,...,n$$ \hspace{1cm} (14)

Step 6: Obtain the closeness coefficient (CC) and rank the order of alternatives.

The $CC_i$ is defined to determine the ranking order of all possible alternatives once the $d_i^+$ and $d_i^-$ of each alternative have been calculated. The closeness coefficient represents the distances to the fuzzy positive ideal solution $A^+$ and $A^-$ simultaneously and is calculated as:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \hspace{1cm} i=1,2,...,m$$ \hspace{1cm} (15)

According to the $CC_i$, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives. It is clear that $CC_i = 1$ if $A_i = A^+$ and $CC_i = 0$ if $A_i = A^-$. Some fuzzy TOPSIS methods were developed in the different applied field. Lin and Chang [16] adopted fuzzy TOPSIS for order selection and pricing of manufacturer (supplier) with make-to-order basis when orders exceed production capacity. [7] extended the TOPSIS method based on interval-valued fuzzy sets in decision analysis. [1] used interval-valued fuzzy TOPSIS method is aiming at solving MCDM problems in which the weights of criteria are unequal, using interval-valued fuzzy sets concepts. [17] designed a model of TOPSIS for the fuzzy environment with the introduction of appropriate negations for obtaining ideal solutions., [6] identified the strategic main and sub-criteria of alliance partner selection that companies consider the most important through Fuzzy AHP and fuzzy TOPSIS model and achieved the final partner-ranking results. [21] applied fuzzy TOPSIS to help the Air Force
developed a compromise ratio (CR) methodology for fuzzy multi-attribute group decision making (FMAGDM), which is an important part of decision support system. [23] generalized TOPSIS to fuzzy multiple-criteria group decision-making (FMCGDM) in a fuzzy environment. [13] proposed a fuzzy hierarchical TOPSIS model for the multi-criteria evaluation of the industrial robotic systems. [4] presented a fuzzy TOPSIS approach for evaluating dynamically the service quality of three hotels of an important corporation in Grand Canaria island via surveys. [22] proposed a fuzzy TOPSIS method based on alpha level sets and presents a non-linear programming solution procedure. [6] applied fuzzy TOPSIS approach to deal with the supplier selection problem in supply chain system.

The algorithm of the Fuzzy TOPSIS Method can be summarized as follows [7]:

1. A committee of decision-makers is formed and the evaluation criteria are determined.
2. Appropriate linguistic variables for the importance weight of the criteria and linguistic ratings for alternatives are determined by decision makers.
3. The weights of criteria aggregated to get the total fuzzy weight $w_i$ of the criterion $C_j$ and decision makers’s ratings are gathered to get the aggregated fuzzy rating $x_{ij}$ of alternative $A_i$ under criterion $C_j$.
4. Fuzzy decision matrix and normalized fuzzy decision matrix are formed.
5. Weighted normalized fuzzy decision matrix is formed.
6. Fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are determined.
7. The distance of each alternative from PIS and FNIS are calculated.
8. The closeness coefficient of each alternative is calculated.
9. By comparing the closeness coefficient, order of all alternatives can be determined.

4. Personnel Selecting Using Fuzzy TOPSIS Approach

In this research, six experts and managers were invited to survey five alternatives and through the literature investigation and experts’ opinions, the committee finally adopted five main criteria. This research includes five evaluation criteria, such as Work experience (C1), Analytical thinking (C2), Foreign language (C3), Working in teams (C4) and Bachelor and Master Degree (C5). In addition, there are five alternatives include: person number one (A1), (A2), (A3), (A4) and (A5). We assume that questionnaire have collected completely and will start with building dataset that are collected. The evaluators have their own range for the linguistic variables employed in this study according to their subjective judgments (Hsieh, Lu, & Tzeng, 2004). For each evaluator with the same importance, this study employs the method of average value to integrate the fuzzy/vague judgment values of different evaluators regarding the same evaluation dimensions. The evaluators then adopted linguistic terms shown in Table I to express their opinions about the rating of criteria and the results are shown in Table 3. Then, the evaluators use the linguistic variables in Table 2 to evaluate the
ratings of alternatives with respect to each criterion and their ratings under five criteria are shown in Table 4.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Decision Makers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D1</td>
</tr>
<tr>
<td>C1</td>
<td>VH</td>
</tr>
<tr>
<td>C2</td>
<td>VH</td>
</tr>
<tr>
<td>C3</td>
<td>H</td>
</tr>
<tr>
<td>C4</td>
<td>VH</td>
</tr>
<tr>
<td>C5</td>
<td>H</td>
</tr>
</tbody>
</table>

Table 3 Importance Weights of Criteria From Six Decision Makers

Table 4. Ratings of The Alternatives by Decision Makers Under The Criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Alternatives</th>
<th>Decision Makers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
<td>A2</td>
</tr>
<tr>
<td>C1</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>A2</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>A3</td>
<td>MG</td>
<td>G</td>
</tr>
<tr>
<td>A4</td>
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<td>G</td>
</tr>
<tr>
<td>A5</td>
<td>VG</td>
<td>VG</td>
</tr>
<tr>
<td>C2</td>
<td>A1</td>
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<tr>
<td>A1</td>
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<tr>
<td>C3</td>
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</tr>
<tr>
<td>A2</td>
<td>F</td>
<td>MP</td>
</tr>
<tr>
<td>A3</td>
<td>VG</td>
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<tr>
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<td>A1</td>
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<tr>
<td>A5</td>
<td>VG</td>
<td>VG</td>
</tr>
</tbody>
</table>

Table 4. Ratings of The Alternatives by Decision Makers Under The Criteria

Then linguistic variables shown in Tables 3 and 4 are converted into triangular fuzzy numbers to form fuzzy decision matrix as shown in Table 5.
The normalized fuzzy decision matrix is formed as in Table 6 and the weighted normalized fuzzy decision matrix is formed as in Table 7.

After forming weighted normalized fuzzy decision matrix FPIS and FNIS are determined as:

\[
\begin{align*}
\mathbf{A}^+ &= [(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1)] \\
\mathbf{A}^- &= [(0,0,0),(0,0,0),(0,0,0),(0,0,0)]
\end{align*}
\]

Then the distance of each alternative from FPIS and FNIS with respect to each criterion are calculated. The results of all alternatives’ distances from FPIS and FNIS and their closeness coefficients are shown in Table 8.
According to the closeness coefficient, the best alternative is the alternative 4 as its closeness coefficient has the highest value. In other words, the fourth alternative is closer to the FPIS and farther from the FNIS.

5. Conclusion

Decision making problem is the process of finding the best option from all of the alternatives and personnel selection is a very good example for this type of problem. In the case of uncertainty, fuzzy theory can be used to solve it. In this paper Fuzzy TOPSIS method has been proposed and the fourth alternative is determined as the best alternative which has the highest coefficient. The distance between two fuzzy triangular numbers is calculated with vertex method but other method like Euclidean can be used in calculating. Further applications. In future studies, other multi criteria methods like Promethee, Vikor can be used to solve the other type of personnel selection problems.
References


